Realistic Friction Rendering using Time-Varying Friction Coefficients

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Abstract—In this study, we propose time-varying friction coefficients, which was time-constant in conventional haptic renderings. The proposed rendering reproduces realistic stickslip vibrations. We implemented it with a haptic interface SPIDAR and evaluated effectiveness of the proposed method by comparing friction sensation in virtual and real worlds.

I. INTRODUCTION

In the real world, friction is dominant force between contacts. So, haptic rendering of friction force is important for virtual reality and haptic interaction.

Zilles [1] integrate haptic rendering of friction for force displays into his GOD object method. His method treats static and dynamic friction and changing of friction force caused by the transitions. Konyo *et al.* [2] proposed to render friction by vibrations caused by stick-slip phenomena. They model the finger tip with a mass spring damper system and friction transition at the contact point. Their model reproduce stick-slip phenomena and each state transtion generates a vibration of damped sinusoid.

Ikeda *et al.* [3] added the vibration feedback in [2] into GOD object method [1] by adding a mass property to the GOD object and a damper between the finger position and the GOD object in addition to the spring. This method enhances reality of the friction a little. In this paper we propose to use time-varying friction coefficients for friction model for Ikeda *et al.*'s method. We confirm that the timevarying friction coefficients reproduce the wave form of the tangential feedback force of stick-slip phenomena in the real world. A user study also shows that the reality of the friction rendered by a force display is improved.

II. FRICTION RENDERING MODEL

Figure 1 shows the haptic rendering model in [3], which combines the stick-slip vibration model used in [2] and GOD object method method in [1].



Fig. 1. Haptic rendering model to reproduce stick-slip

¹Laboratory for Future Interdisciplinary Research of Science and Technology, Institute of Innovative Research, Tokyo Institute of Technology, Yokohama, Japan info@haselab.net In the haptic rendering, transitions between static and dynamic frictions occur such as in [2]. The stick-to-slip transition occurs when

$$|m\ddot{x_0} + b(\dot{x_0} - \dot{x}) + k(x_0 - x)| > \mu_s F_N = \mu_s ky, \quad (1)$$

where m and x_0 are the mass and the position of the GOD object, b and k are damping and spring coefficient, x is finger position, F_N is normal force and μ_s is static friction coefficient. In this moment, the finger and the object would be oscillated corresponding to the stored elastic displacement. To reproduce this oscillation, a damped sinusoid Q(t) is added to tangential feedback force;

$$Q(t) = A\mu_s F_N(t_0) \exp(-D(t-t_0)) \frac{1}{v} \sin(v\omega_0(t-t_0))$$
(2)

where A and D are coefficients for oscillation amplitute and damping, t_0 is the time of slipping and v is velocity of the finger.

The slip-to-stick transition occurs when

$$|m\ddot{x_0} + b(\dot{x_0} - \dot{x}) + k(x_0 - x)| < \mu_d F_N = \mu_d ky \quad (3)$$

where μ_d is dynamic friction coefficient.

III. TIME-VARYING FRICTION COEFFICIENTS

For the friction coefficients in the previous section, conventional works use time constant coefficients. In this section, we propose to use time-varying friction coefficients and show the effect on the state transition and resultant vibration in the next section.

In the real world, friction coefficients are time-varying parameters. Dieterich [4] empirically models time dependency as

$$\mu = \mu_0 + Alog(Bt+1) \tag{4}$$

where mu_0, A, B are constants, t is time of contact (duration of stick state). Regarding equation 4 we model static friction coefficient μ_s as

$$\mu_s = \mu_d + B \log_{10}(Ct + 1) \tag{5}$$

where B and C are constants and μ_d is dynamic friction coefficient.

IV. EXPERIMENT

A. Experimental implementation

Figure 2 shows experimental environment, where updated SPIDAR [5] is used as a haptic interface to provide both the feedback force and the vibration. The virtual world has a floor and a spherical haptic pointer. The simulation, haptic rendering and the device control are run at 1kHz. Equations

discretized by explicit Euler method are used for simulation. The constants for time-varying friction coefficients in Equation 5 were $\mu_d = 0.6, B = 0.3, C = 500$. The constants for vibration in Equation 2 were A = 50 and D = 0.04. Time constant friction coefficients for comparison were $\mu_s = 1.0$ and $\mu_d = 0.6$.



Fig. 2. Haptic rendering model to reproduce stick-slip

B. Subjective evaluation

Six participants are asked to slide their finger on the desk for one minute. Then they are asked to slide haptic pointer in the virtual world to compare the frictions in time-constant and time-varying conditions. The order of the presentations is randomized to avoid order effect. Then they choose an answer from "friction in one of the conditions is near to the real one" and "there is no difference". As the results, all six participants choose the time-varying condition.

C. Comparison of friction forces

Acceleration of a finger is measured while sliding at 0.03[m/s] and friction force in the real world is estimated with an equation of motion of $ma = F_e - F_f$, where m is mass of hand set to 1[kg], a is acceleration, F_e is the external force added to the finger and F_f is the friction force. The external force is set as $F_e = 0$ for t < -0.15, 6 + 40t[N] for -0.15 < t < 0, 4[N] for $t \ge 0$, where t = 0[s] is time of sliding.

Figure 3 shows the measured acceleration and the estimated friction force for a stroke in the real world. Figure 4 shows rendered friction force by the time-constant friction coefficients and time-varying friction coefficients.

The stick-slip oscillation in the estimated friction force in the real world (Fig. 3) decreases from 0.2 [s] to 0.7[s] and stops. After 0.7[s], only high frequency noises remain. These features are reproduced in rendered force by timevarying model (Fig. 4 bottom) while time-constant model continuously generate stick-slip oscillations (Fig. 4 top).

V. CONCLUSIONS

In this study, we propose to introduce time-varying friction coefficients into haptic rendering of friction force. The experimental results show that the proposed rendering reproduces more realistic sliding friction sensations and waveforms of friction forces compare to conventional rendering with timeconstant friction coefficients.





Fig. 4. Friction forces rendered by time-constant and time-varying model

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